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Updated bounds on milli-charged particles

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Abstract: We update the bounds on fermions with electric charge ϵe and mass m_ϵ. For $m_\epsilon \lesssim m_e$ we find $10^{-15} \lesssim \epsilon < 1$ is excluded by laboratory experiments, astrophysics and cosmology. For larger masses, the limits are less restrictive and depend on m_ϵ. For milli-charged neutrinos, the limits are stronger, especially if the different flavors mix as suggested by current experimental evidence.

Keywords: Cosmology of Theories beyond the SM, Related Astrophysics (Neutron Stars, Supernovae etc.), Beyond Standard Model.

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Contents

1. Introduction

A puzzle of continuing interest in particle physics is the apparent quantization of electric charge. The electric charge of all the particles we see appears [1] to be an integer multiple of $Q_e/3$, where Q_e is the electron charge. However it is theoretically consistent to have particles with electric charge ϵQ_e where ϵ is any real number. We consider $\epsilon < 1$ in this paper, and refer to these particles as "milli-charged."

Milli-charged particles can be introduced into the Standard Model in a variety of ways. One is to add an $\mathrm{SU}(3)\times\mathrm{SU}(2)$ singlet Dirac fermion with hypercharge $Y = 2\epsilon$. However, this is not simple [2] if the hypercharge $\mathrm{U}(1)_Y$ is embedded in a grand unified gauge group. A second way of generating effectively milli-charged particles, which works even if hypercharge is quantised, is due to Holdom [3]. He introduced a second unbroken "mirror" $\mathrm{U}(1)'$, and showed that the photon and the "paraphoton" can mix, so that a particle charged under the $\mathrm{U}(1)'$ appears to have a small coupling to the photon. A third mechanism for introducing milli-charged particles is to remain with the Standard Model particle content and allow the neutrinos to have small electric charges [4]. If the Standard Model hypercharge operator Y_{SM} is redefined to be $Y' = Y_{SM} + 2\sum_i \epsilon_i (B/3 - L_i)$, neutrinos acquire small electric charges ϵ_i and the Standard Model anomaly cancellation is preserved. We assume here that $\mathrm{U}(1)_{em}$ is an unbroken symmetry; it can be spontaneously broken in models with milli-charged scalars [20], but the bounds in this case are slightly different from what we discuss here [21] (for instance the photon has a mass).

The purpose of this paper is to collect and update experimental and observational bounds on milli-charged particles. Such limits have been previously considered for milli-charged fermions that are not neutrinos [6]–[14] and for milli-charged neutrinos [5, 15, 16, 17, 18]. See also [19] for a discussion of astrophysical constraints on all types of milli-charged particles. We recalculate the laboratory bounds using recent data, and include the limits from recent experiments searching for milli-charged particles [27, 28]. We numerically evaluate the constraint from Big Bang Nucleosynthesis on particles with milli-hypercharge, and find a bound similar to previous estimates. We also revisit the astrophysical constraints, resolving a discrepancy in the White Dwarf bounds between [9] and [12] (in favour of the stronger bound of [9]). Our Supernova limit differs slightly from what is found in ref. [10], as will be explained later, and the Red Giant bound remains unchanged. We quote the previous estimated bounds [12] from balloon experiments searching for strongly interacting Dark Matter, and from underground WIMP detectors. For a more detailed discussion of the calculations, see for instance [12, 14, 17, 19].

Our conclusions are presented in figure 1 which is an update of ref. [14, figure 1]. It presents bounds on milli-charged fermions in models with an extra $\mathrm{U}(1)'$, and without. Milli-charged neutrinos will be briefly discussed at the end of the paper. The limits from laboratory experiments, from Red Giants and White Dwarfs are independent of whether there is a paraphoton, and appear as solid lines. We discuss the Supernova and nucleosynthesis bounds with some care for the model without a paraphoton; the limits should be similar in the presence of a paraphoton [14] so these bounds also appear as solid lines. The limit from $\Omega < 1$ differs in the two models: in the absence of a paraphoton, the millicharge annihilation cross-section is proportional to ϵ^2/m^2. So the excluded area, indicated with dotted lines, extends to lower masses for smaller ϵ. The bound on models with a paraphoton is the horizontal dashed line.

2. Laboratory bounds

We plot four lines on figure 1 from laboratory data: 1) a combined “accelerator” line consisting of the limits from LEP and beam dump experiments [7, 12],[1] 2) the bound found by the Tokyo group [27] from the non-observation of invisible Orthopositronium decay, 3) the limit from the dedicated milli-charged particle search experiment at SLAC [28], and 4) an updated constraint from the Lamb shift using more recent data [24].

[1] The values of m_ϵ excluded by direct searches are quoted in table II of [12], but the values given are in MeV, not GeV as claimed in the caption. The limit is placed correctly, however, in [12, figure 2].

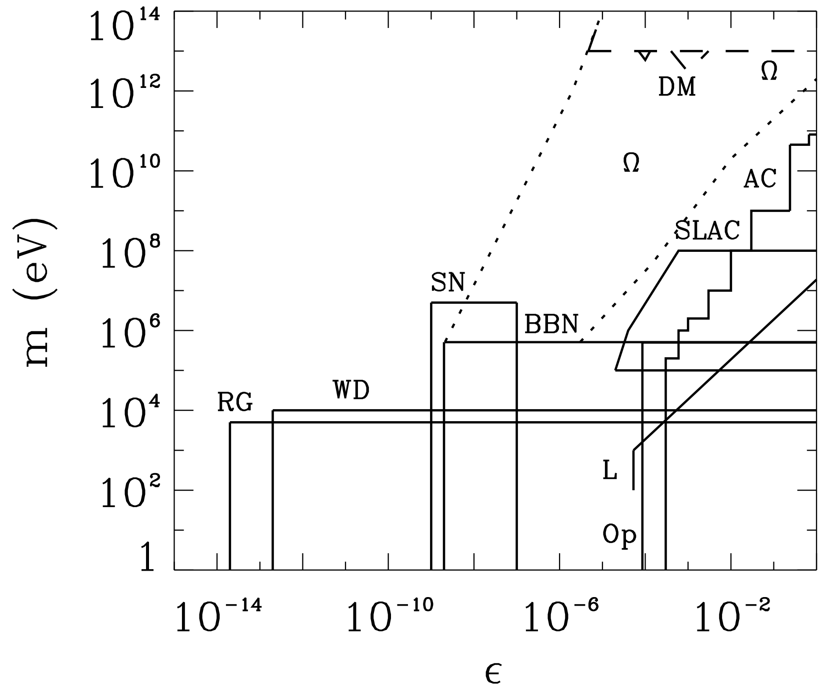

Figure 1: Regions of mass-charge space ruled out for milli-charged particles. The solid and dashed lines apply to the model with a paraphoton; solid and dotted lines apply in the absence of a paraphoton. The bounds arise from the following constraints: AC — accelerator experiments; Op — the Tokyo search for the invisible decay of ortho-positronium [27]; SLAC — the SLAC milli-charged particle search [28]; L — the Lamb shift; BBN — nucleosynthesis; Ω — $\Omega < 1$; RG — plasmon decay in red giants; WD — plasmon decay in white dwarfs; DM — dark matter searches; SN — Supernova 1987A.

LEP has taken many years of data since limits from LEP were previously considered [12], so we briefly discuss possible bounds, although these are weak because the milli-charge coupling to the Z is suppressed by $\sin^2\theta_W$. A search for particles with fractional charge $\epsilon = 2/3$ was performed by OPAL using 1991-93 data [22], which rules out $\epsilon \geq 2/3$ for $m_\epsilon < 84\,\mathrm{GeV}$. This bound could be extended to the present kinematic limit $m_\epsilon < 100\,\mathrm{GeV}$, if one assumes that a particle with $1 > \epsilon > 2/3$ would be seen as such in the detector. Fractionally charged particles would contribute to the invisible width of the Z if they were not seen in the detector,[2] in which case the LEP bound can be extended to

$$\epsilon < 0.24 \qquad m_\epsilon > 45\,\mathrm{GeV}\,, \tag{2.1}$$

from requiring that that milli-charges not contribute more than the 2σ error to the invisible width of the Z at LEP1.

[2]This bound assumes that particles with $1/4 < \epsilon < 2/3$ would look like noise in the detector, and not be mis-identified as $\epsilon = 1$ tracks.

We calculate an improved bound from the Lamb shift measurements by requiring that the milli-charged particle vacuum polarisation contribution be less than the 2σ error ($= 20 \times 10^{-3}$ MHz) [24]. This is a more recent measurements of the $2S_{1/2} - 2P_{1/2}$ Lamb shift [24] than used in [12]. There are more precise measurements of combinations of Lamb shifts of different principal quantum number n ($n \neq 2$) [25]; however they do not substantially improve our bound so we use the $2S_{1/2} - 2P_{1/2}$ results because the milli-charged contribution is easier to disentangle. The bound from the Lamb shift is stronger than from $g-2$, despite the fact that $g-2$ is measured very precisely [26], because milli-charged particles contribute to the Lamb shift at one loop but to $g-2$ at two loop. For a discussion of the effects of milli-charged particles on precision QED measurements, see [9, 23].

3. Big-bang nucleosynthesis

3.1 Dominant processes

Particles with small electric charge will interact with the plasma in the early universe and in this way get thermally excited. Therefore, their primordial energy density and thus their interaction strength can be constrained by the standard nucleosynthesis limit on $N_{\rm eff}$, the effective number of thermally excited neutrino degrees of freedom. A milli-charged fermion must be a Dirac particle and thus has four intrinsic degrees of freedom. If it were fully excited in the early universe, it would contribute $\Delta N_{\rm eff} = 2$ prior to the annihilation of electrons and positrons.[3] Such a particle would be excluded by a large margin. During this annihilation process entropy is transferred to the milli-charged particles, heating them relative to the neutrinos. At the same time, the photon temperature, T_γ, relative to the neutrino temperature, T_ν, is less than in the standard model. Normally, $T_\nu/T_\gamma \simeq 0.714$, whereas here $T_\nu/T_\gamma \simeq 0.849$. Altogether, neutrinos plus milli-charged particles contribute $N_{\rm eff} = 3 \times (0.849/0.714)^4 + 2 \times 1.401^4 = 13.7$, so that $\Delta N_{\rm eff} = 10.7$ at late times.

While it is easy to estimate $\Delta N_{\rm eff}$ for milli-charged particles where full thermal equilibrium obtains, we can derive a more restrictive limit by studying its value as a function of ϵ in a regime where full equilibrium is not achieved. To this end we assume that the new particles interact only electromagnetically by their milli-charge. The bounds in the case where there is an extra U(1) are at least as restrictive, due to the presence of the paraphoton [14].

We find that the dominant electromagnetic processes which couple the gas of milli-charged particles f to the electromagnetic plasma are

$$e^+e^- \to f\bar{f}\,, \qquad \gamma \to f\bar{f}\,, \qquad ef \to ef\,. \tag{3.1}$$

[3] This is when the BBN bound on the number of neutrinos applies.

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The rates for these reactions are proportional to ϵ^2 while those for other conceivable processes such as $\gamma\gamma \to f\bar{f}$ or $\gamma f \to \gamma f$ are proportional to ϵ^4 and thus much smaller. At low temperatures, the electron-positron number density is exponentially suppressed, making the $f\bar{f}$ production rate approach zero quickly when $T \lesssim m_e$. The third process in eq. (3.1) does not contribute to the production of milli-charged particles but helps to achieve kinetic equilibrium.

For a calculation of the $f\bar{f}$ production rates we use Boltzmann statistics for all species. Then the rate for the pair process is [38] $\Gamma_{e^+e^-\to f\bar{f}} = n_e\langle\sigma|v|\rangle$, where $\langle\sigma|v|\rangle$ is the thermally averaged annihilation cross section. It can be expressed as [36]

$$\langle\sigma|v|\rangle = \frac{1}{8m_e^4 T K_2^2(m_e/T)} \int_{4m_e^2}^{\infty} ds\, s^{1/2}(s-4m_e^2) K_1\left(\frac{s^{1/2}}{T}\right) \sigma_{\rm CM}\,, \tag{3.2}$$

where K_n is a modified Bessel function of the second kind and $\sigma_{\rm CM}$ is the center-of-mass cross section. The squared matrix element is

$$\frac{1}{4}\sum |M|^2 = \frac{8\epsilon^2 e^4}{(p_{e^-}+p_{e^+})^4}\left[(p_{e^-}p_f)(p_{e^+}p_{\bar{f}}) + (p_{e^-}p_{\bar{f}})(p_{e^+}p_f) + m_f^2(p_{e^-}p_{e^+}) + \right.$$
$$\left. + m_e^2(p_f p_{\bar{f}}) + 2m_e^2 m_f^2\right], \tag{3.3}$$

which translates into the usual CM cross section $\sigma_{\rm CM} = (4\pi/3)\,\epsilon^2\alpha^2\, s^{-1}$ if all the particles can be considered massless. In this limit we find

$$\langle\Gamma_{e^+e^-\to f\bar{f}}\rangle = \frac{\zeta_3}{2\pi}\,\epsilon^2\alpha^2 T\,, \tag{3.4}$$

where ζ refers to the Riemann zeta function.

The decay rate of a transverse plasmon (photon) with energy ω and momentum k into massless milli-charged particles is $\Gamma_{\gamma\to f\bar{f}} = \epsilon^2\alpha Z(\omega^2-k^2)/3\omega$ [19]. In a relativistic plasma, the plasma frequency is given by $\omega_{\rm P}^2 = (4\pi/9)\alpha T^2$. Except for the low-energy part of the blackbody photon spectrum we have $\omega \gg \omega_{\rm P}$, a limit where the dispersion relation is $\omega^2 - k^2 = \frac{3}{2}\omega_{\rm P}^2$ and the wave-function renormalization factor is $Z = 1$ [19]. With Boltzmann statistics we find $\langle\omega^{-1}\rangle = (2T)^{-1}$ so that finally the average plasmon decay rate is

$$\langle\Gamma_{\gamma\to f\bar{f}}\rangle = \frac{\pi}{9}\,\epsilon^2\alpha^2 T. \tag{3.5}$$

Comparing the rates for the two processes we find

$$\frac{\langle\Gamma_{e^+e^-\to f\bar{f}}\rangle}{\langle\Gamma_{\gamma\to f\bar{f}}\rangle} = \frac{9\zeta_3}{2\pi^2} = 0.55, \tag{3.6}$$

implying that the plasma process is actually somewhat more important than pair annihilation. Notice, however, that the plasma process is only important if $m \lesssim \omega_P/2$. During BBN, the plasma frequency is roughly given by $\omega_P \simeq \sqrt{(4\pi/9)\alpha}T \simeq$

$0.1\,T$, so that at $T = 1\,\mathrm{MeV}$, where BBN begins, the plasma process is only important for $m \lesssim 50\,\mathrm{keV}$. Therefore the plasma process is important only for milli-charged particles with masses in the keV region, where astrophysical bounds from stellar evolution are much more stringent. Thus, it seems safe no neglect the plasma process in the BBN calculations.

3.2 Solving the Boltzmann equation

The standard procedure for calculating the evolution of number and energy density of a given species is to use the Boltzmann collision equation. It describes the time evolution of the single particle distribution function, f_1, of any given species. In the expanding universe, formally this equation can be written in our case as [38]

$$\frac{\partial f_1}{\partial t} - Hp\,\frac{\partial f_1}{\partial p} = (C_{\mathrm{ann}} + C_{\mathrm{el}})\,[f_1]\,, \tag{3.7}$$

where $H \equiv \dot{R}/R$ is the Hubble expansion parameter. On the right-hand side, C_{ann} is the collision operator describing pair annihilation. C_{el} is the elastic-scattering term which includes the combined effect of scattering on electrons, positrons, and photons.

The e^+e^- process is of the form $1 + 2 \to 3 + 4$ so that the collision term can be written generically as [38]

$$C_{\mathrm{ann}}[f_1] = \frac{1}{2E_1}\int d^3\tilde{p}_2 d^3\tilde{p}_3 d^3\tilde{p}_4 \Lambda(f_1, f_2, f_3, f_4) \sum |M|^2 \delta^4(p_1 + p_2 - p_3 - p_4)(2\pi)^4\,. \tag{3.8}$$

Here, $d^3\tilde{p} \equiv d^3p/[(2\pi)^3 2E]$, $\sum |M|^2$ is the squared matrix element, and p_i is the four-momentum of particle i. The phase-space factor is $\Lambda \equiv f_3 f_4(1 - f_1)(1 - f_2) - f_1 f_2(1 - f_3)(1 - f_4)$.

The dominant elastic scattering term from scattering on electrons and positrons suffers from the usual infrared Coulomb divergence. It can be regulated by including Debye screening effects in the medium. However, instead of treating elastic scattering in any detail we calculate the evolution of the milli-charged particle ensemble for two extreme cases, (a) No elastic scattering, and (b) Elastic scattering is assumed to be efficient enough to bring the milli-charged species into complete kinetic equilibrium at all times. It will turn out that the difference between these two cases is quite small, justifying our neglect of a detailed treatment of elastic scattering.

The dynamical evolution of the cosmic scale factor is governed by the Friedmann equation and the equation of energy conservation [38],

$$\begin{aligned} H^2 &= \frac{8\pi G\rho}{3} \\ \frac{d}{dt}(\rho R^3) &= -P\frac{d}{dt}(R^3)\,, \end{aligned} \tag{3.9}$$

where ρ is the energy density and P the pressure, both including the effect of milli-charged particles.

We have solved these equations to find the energy density of milli-charged particles. We have always neglected their mass, an approximation which is accurate for m_ϵ up to the electron mass. For larger masses the milli-charged particles can pair annihilate into electron-positron pairs at low temperatures. We have always taken a zero initial population of milli-charged particles, an assumption which is valid if they interact only via their electric charge.

Adding energy density to the cosmic plasma around the epoch of e^+e^- annihilation perturbs Big Bang Nucleosynthesis [38]. In order to calculate constraints on ϵ we have used the nucleosynthesis code of Kawano [32], modified to include the energy density in milli-charged particles as well as the change in neutrino-to-photon temperature ratio. The effect of the milli-charged particles corresponds to additional neutrino degrees of freedom of

$$\Delta N_{\rm eff} = 0.69 \times 10^{17}\, \epsilon^2 \times \begin{cases} 1 & \text{no elastic scattering,} \\ 1.39 & \text{full equilibrium,} \end{cases} \tag{3.10}$$

an approximation is valid for $\epsilon \lesssim 6 \times 10^{-9}$. This is more than sufficient for our purposes since all values of ϵ higher than this produce such a large $\Delta N_{\rm eff}$ that they are excluded by a huge margin. As expected, the effect of elastic scattering on the energy density in milli-charged particles is quite small as this process does not produce additional particles.

3.3 BBN limits

There are still some unresolved issues regarding the observationally determined values of the primordial light-element abundances. For the past few years there have been two favoured solutions, namely the so-called High-Helium/Low-Deuterium and the Low-Helium/High-Deuterium solutions [33]. There seems to be growing consensus that the first is the correct one. Nevertheless any bound derived from nucleosynthesis should be used with some caution since the question of primordial deuterium and helium abundances is not yet fully settled.

In the present paper we shall use the data on primordial helium obtained by Izotov and Thuan [34] and the deuterium data from Burles and Tytler [35]

$$\begin{aligned} Y_P &= 0.244 \pm 0.002 \\ D/H &= (3.39 \pm 0.25) \times 10^{-5}\,, \end{aligned} \tag{3.11}$$

where errors are estimated 1σ uncertainties. These data give the High-He/Low-D solution and are completely consistent with standard BBN for a baryon-to-photon ratio of $\eta = (5.1 \pm 0.3) \times 10^{-10}$ [35]. Used together, these data provide the constraint

$$N_{\rm eff} = 2.98 \pm 0.33 \;\; (90\%\ \text{C.L.})\,, \tag{3.12}$$

on the effective number of neutrinos, thus tightly constraining any non-standard nucleosynthesis scenario.

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Together with the less restrictive case (no elastic scattering) of eq. (3.10) BBN then implies a limit

$$\epsilon < 2.1 \times 10^{-9} \,. \tag{3.13}$$

This bound applies to masses in the regime $m_\epsilon \lesssim m_e$. While our result is very similar to what one finds from simple dimensional estimates, our calculation is quantitative and the errors are controlled.

4. Stellar evolution

4.1 Globular clusters

New low-mass particles will be produced in the hot and dense medium in the interior of stars and subsequently escape. This new energy-loss channel leads to observational modifications of the standard course of stellar evolution and thus can be used to set limits on the particle's interaction strength. For milli-charged particles, limits have been set from red giants [5, 9, 12, 14, 13, 39], horizontal-branch (HB) stars [19], white dwarfs [9, 12], and supernova (SN) 1987A [10]. For small masses of the milli-charged particles, the most restrictive limits arise from HB stars and low-mass red giants in globular clusters.

At the end of the main-sequence evolution of normal stars, the hydrogen in the inner part has been consumed, leaving the star with a core consisting mainly of helium. In low-mass stars, this helium core reaches degeneracy before it is hot and dense enough to be ignited. This process is very dependent on density and temperature, and even minor changes in these quantities produce observable changes in the brightness at the tip of the red-giant branch in globular clusters. Therefore, the core mass at helium ignition as implied by the color-magnitude diagrams of several globular clusters implies a limit on any new energy-loss channel. After helium ignition, the stars move to the horizontal branch where they burn helium in their core. A new energy-loss mechanism will lead to an accelerated consumption of nuclear fuel, shortening the helium-burning lifetime which can be "measured" by number counts of HB stars in globular clusters. When applied to a new energy-loss channel, usually one of these arguments is more restrictive [19]. For example, a putative neutrino dipole moment will add to the efficiency of the plasmon decay process $\gamma \to \nu\bar{\nu}$ and thus enhance neutrino losses. The helium-ignition argument provides far more restrictive limits than the helium-burning lifetime argument because the plasmon decay is more effective in the degenerate red-giant core. On the other hand, axion losses by the Primakoff process are more effective in the nondegenerate cores of HB stars so that the helium-burning lifetime argument yields more restrictive limits.

The emission of milli-charged particles is a special case in that both arguments yield comparable limits of about [19]

$$\epsilon \leq 2 \times 10^{-14} \,. \tag{4.1}$$

The reason is that the rate of the plasma decay process $\gamma \to f\bar{f}$ for milli-charged particles is proportional to $\omega_{\rm P}^2$, as opposed to the magnetic-dipole case ($\omega_{\rm P}^4$) or standard-model neutrino case ($\omega_{\rm P}^6$). The low power of the plasma frequency implies that the emission rate per unit mass is almost independent of density. This, in turn, implies that the core expansion caused by helium ignition leaves the energy-loss rate per unit mass nearly unchanged, while it is "switched off" for the dipole-moment case, or "switched on" for the axion case.

An average value for the plasma frequency in the core of a globular-cluster star before helium ignition is $\omega_{\rm P} \simeq 10\,{\rm keV}$ (in the center it is about twice that) while it is about $2\,{\rm keV}$ in the core of a HB star. Therefore, the helium-ignition argument constrains milli-charged particles with masses up to about $5\,{\rm keV}$.

Of course, this sort of argument applies only if the interaction strength is small enough that the milli-charged particles escape freely once produced in the stellar core; one can check that this is the case for $\epsilon \lesssim 10^{-8}$ [19]. For larger charges, the particles would contribute to the transfer of energy rather than carrying away energy directly. In order to avoid observable consequences, the efficiency of energy transfer would have to be less than that of photons, i.e. the mean free path would have to be very short. It is unlikely that there exists an allowed range of milli-charges $\epsilon \lesssim 1$ where all stars would be left unchanged. However, since laboratory limits and the BBN argument exclude values for ϵ above 10^{-8} anyway, a detailed discussion of the trapping limit is not warranted.

4.2 White dwarfs

The observed population of hot young white dwarfs is consistent with cooling by surface emission of photons and by volume emission of neutrinos produced by plasmon decay via Standard Model interactions [29, 19]. Blinnikov and Dunina-Barkovskaya [29] set a bound on the neutrino magnetic moment, $\mu_\nu < 10^{-11}\mu_B$ (where $\mu_B = e/2m_e$), by requiring that the additional neutrino emission not cool the white dwarfs faster than observed. We can translate this bound into one on milli-charged particles.

For neutrinos or neutrino-like particles coupling to the photon via a dipole moment, the energy loss rate can be written as

$$Q_\mu \propto \frac{\mu^2}{2}\left(\frac{\omega_{\rm P}^2}{4\pi}\right)^2 Q_2\,, \tag{4.2}$$

whereas for milli-charged particles, it is given by

$$Q_\epsilon \propto \epsilon^2\alpha\,\frac{\omega_{\rm P}^2}{4\pi}\,Q_1\,, \tag{4.3}$$

where Q_1/Q_2 is a factor of order unity [19]. Here, we set $Q_1 = Q_2 = 1$. One then obtains

$$\frac{Q_\epsilon}{Q_\mu} = 2.09\,\frac{\epsilon_{14}^2}{\mu_{12}^2}\left(\frac{\omega_{\rm P}}{10\,{\rm keV}}\right)^{-2}\,, \tag{4.4}$$

where $\epsilon_{14} \equiv 10^{14}\,\epsilon$ and $\mu_{12} \equiv 10^{12}\,\mu/\mu_B$. In order to calculate an average value of this ratio over the entire star, we need to perform an "emissivity-average" of the form

$$\left\langle \frac{Q_\epsilon}{Q_\mu} \right\rangle = \frac{\int drr^2 \frac{Q_\epsilon}{Q_\mu} Q_\mu}{\int drr^2 Q_\mu}\,. \tag{4.5}$$

In order to do these integrals, we assume that the white dwarf is a polytrope of index $n = 3/2$, so that

$$\left\langle \frac{Q_\epsilon}{Q_\mu} \right\rangle = 2.40 \times 10^5 \rho_{c,6}^{-1} \frac{\epsilon_{14}^2}{\mu_{12}^2} \frac{\int d\xi \xi^2 \theta^n}{\int d\xi \xi^2 \theta^{2n}}\,, \tag{4.6}$$

where $r = \alpha\xi$ and $\rho = \rho_c \theta^n$ [37]. For a white-dwarf mass of 0.7 solar masses, this gives

$$\left\langle \frac{Q_\epsilon}{Q_\mu} \right\rangle = 0.34\, \frac{\epsilon_{14}^2}{\mu_{12}^2}\,. \tag{4.7}$$

If we then demand that the emission rate due to milli-charges is not larger than for neutrino dipole moments, we find

$$\epsilon_{14} \leq 17\,. \tag{4.8}$$

This bound is consistent with that found in ref. [9].

One may worry that the way the emissivity-average is performed has consequences for the bound. One could instead use

$$\left\langle \frac{Q_\epsilon}{Q_\mu} \right\rangle = \frac{\int drr^2 \frac{Q_\epsilon}{Q_\mu} Q_\epsilon}{\int drr^2 Q_\epsilon}\,. \tag{4.9}$$

This yields $\epsilon_{14} \leq 15$ so that the way the average is performed matters little for the final result.

The emissivity-averaged plasma frequency is $\langle (10\,\mathrm{keV}/\omega_\mathrm{P})^2 \rangle = 0.16$ so that $\langle \omega_\mathrm{P} \rangle = 25\,\mathrm{keV}$. Therefore, the white-dwarf bound applies to milli-charged particles with $m_\epsilon \lesssim 10\,\mathrm{keV}$, similar to the red-giant case.

4.3 Supernova 1987A

Finally, the stellar energy-loss argument can be applied to SN 1987A where the number of neutrinos detected at Earth agree roughly with theoretical expectations. If there are other particles contributing to the cooling of the proto neutron star, this will reduce the neutrino fluxes and the duration of the neutrino signal. Therefore, if we assume that such hypothetical particles freely stream from the core where they are produced, one can put an approximate bound on the allowed loss rate [19] of

$$\langle Q/\rho \rangle \lesssim 10^{19}\ \mathrm{erg\ g^{-1}\ s^{-1}}\,, \tag{4.10}$$

to be calculated at average core conditions of about 3×10^{14} g cm^{-3} and 30 MeV for density and temperature, respectively. 10^{19} erg g^{-1} s^{-1} is the proto-neutron star's average rate of energy loss to neutrinos.

The main production process for milli-charged particles in a SN core is the plasmon process. In a degenerate, relativistic electron plasma it is orders of magnitude larger than e^+e^- annihilation. The energy-loss rate is [19]

$$Q_{\mathrm{P}} = \frac{8\zeta_3}{9\pi^3}\,\epsilon^2\alpha^2\left(\mu_e^2 + \frac{\pi^2 T^2}{3}\right) T^3\, Q_1\,, \tag{4.11}$$

where Q_1 is again a factor of order unity. Setting $Q_1 = 1$, one finds

$$\epsilon \leq 1 \times 10^{-9}\,. \tag{4.12}$$

This bound matches the one found by Mohapatra and Rothstein [10]. However, they considered nucleon-nucleon bremsstrahlung as a production process and notably an amplitude where the electromagnetic current is coupled to an intermediate charged pion. We believe that a naive perturbative calculation of this process in a nuclear medium can be unreliable [19]. However, since the bounds are so similar, a detailed study of the nucleon process is not warranted.

If ϵ is much larger than our limit, the milli-charged particles are trapped inside the proto neutron star, and can only escape via diffusion. The main process which keeps milli-charged particles trapped is Coulomb scattering on protons.[4] The differential scattering cross section on nonrelativistic protons is

$$\frac{d\sigma}{d\Omega} = 2\epsilon^2\alpha^2\,\frac{E^2(1+\cos\theta)}{|\mathbf{q}|^4}\,, \tag{4.13}$$

where $\mathbf{q}$, with $|\mathbf{q}|^2 = 2E^2(1-\cos\theta)$, is the momentum transfer, E the milli-charged particle's energy, and θ the scattering angle. This cross section is strongly forward peaked and thus not a good measure for particle trapping: a particle which is deflected by a small angle continues its way out of the star essentially as if it had not been scattered at all. Therefore, we rather consider the usual *transport* cross section which includes an additional weight $(1-\cos\theta)$. Therefore,

$$\frac{d\sigma_{\mathrm{T}}}{d\Omega} = \epsilon^2\alpha^2\,\frac{(1+\cos\theta)}{\mathbf{q}^2} \tag{4.14}$$

is a more adequate measure for the effectiveness of particle trapping.

In addition, we need to include proton-proton correlations induced by their mutual Coulomb repulsion, i.e. we need to include screening effects. This is achieved by multiplying the cross section with the static structure function $S(\mathbf{q}) = \mathbf{q}^2/(\mathbf{q}^2 + k_{\mathrm{S}}^2)$ so that finally

$$\frac{d\sigma_{\mathrm{T,eff}}}{d\Omega} = \epsilon^2\alpha^2\,\frac{(1+\cos\theta)}{\mathbf{q}^2 + k_{\mathrm{S}}^2}\,. \tag{4.15}$$

[4]The scattering on electrons was instead considered in [10], but this is less important than protons.

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For nonrelativistic, nondegenerate protons, the screening scale k_S^2 is given by the proton Debye scale $k_S^2 = k_D^2 = 4\pi\alpha n_p/T$ with n_p the proton density. It would have been incorrect to use the electron screening scale since the background of degenerate electrons is much "stiffer" than the protons, i.e. most of the polarization of the plasma by a test charge is due to the protons. With these modifications we find a total cross section

$$\sigma_{\mathrm{T,eff}} = \frac{2\pi\epsilon^2\alpha^2}{E^2}\left[\frac{(2+z)}{2}\ln\left(\frac{2+z}{z}\right) - 1\right], \tag{4.16}$$

where

$$z \equiv \frac{k_D^2}{2E^2} = \frac{2\alpha}{3\pi}\eta_e^3\left(\frac{T}{E}\right)^2 = 0.335\left(\frac{\eta_e}{6}\right)^3\left(\frac{T}{E}\right)^2 \tag{4.17}$$

and $\eta_e = \mu_e/T$ is the electron degeneracy parameter. We have used that $n_p = n_e = \mu_e^3/3\pi^2$. The transport mean free path is then found to be

$$\lambda_{\mathrm{T,eff}}^{-1} \equiv \sigma_{\mathrm{T,eff}} n_p = \epsilon^2\alpha T\, z\left[\frac{(2+z)}{2}\ln\left(\frac{2+z}{z}\right) - 1\right], \tag{4.18}$$

where we have expressed n_p in terms of k_D. Taking $T = 30\,\mathrm{MeV}$ as a typical value, and using 1 for the z-dependent expression, we find that $\lambda_{\mathrm{T,eff}}$ exceeds the proto-neutron star radius of about 10 km for $\epsilon \lesssim 1 \times 10^{-8}$.

When ϵ is larger than this, the particles no longer freely escape. Rather, a "photo-sphere" for the milli-charged particles is created. For them to be less effective at carrying away energy, their transport cross section must be about as large as that for neutrinos or larger. Very crudely, we must compare eq. (4.16) with the weak-interaction transport cross section for neutral-current scattering on nucleons,

$$\sigma_{\mathrm{T,weak}} = \frac{2G_F^2E^2}{3\pi}(C_V^2 + 5C_A^2), \tag{4.19}$$

where $|C_A| \approx 1.26/2$ for protons and neutrons, while $|C_V| \approx 0$ for protons and 1/2 for neutrons. If we use $z = 0.01$ near the neutrino sphere, the expression in square brackets in eq. (4.16) is about 4. With this value we find

$$\left(\frac{\sigma_{\mathrm{T,eff}}}{\sigma_{\mathrm{T,weak}}}\right)^{1/2} \approx 1.2 \times 10^7\,\epsilon\left(\frac{20\,\mathrm{MeV}}{E}\right)^2. \tag{4.20}$$

Taking an average energy of 20 MeV we conclude that ϵ should exceed approximately 8×10^{-8} for emission of milli-charged particles to be less important than neutrinos.

While this derivation is somewhat crude, we conclude that milli-charged particles in the range $10^{-9} \lesssim \epsilon \lesssim 10^{-7}$ are excluded. The plasma frequency in a proto-neutron star is roughly 10 MeV so that this bound applies to milli-charged particles with mass below about 5 MeV. The upper bound on ϵ will also apply to the model with a second U(1). The trapping limit $\epsilon > 10^{-7}$ could be slightly modified by the presence of the paraphoton, but this area of parameter space is already ruled out by nucleosynthesis, so we do not discuss this further. In summary, SN 1987A excludes only a very narrow sliver of parameter space in addition to what is excluded by other arguments (figure 1).

5. Milli-charged neutrinos

In the Standard Model with massless neutrinos, there are four anomaly-free U(1) symmetries, corresponding to the Standard Model hypercharge Y_{SM}, and $\{B/3 - L_i\}$. B is baryon number, and L_i are the lepton numbers of the three lepton families. The hypercharge operator can be redefined to be

$$Y' = Y_{\mathrm{SM}} + 2\sum_i \epsilon_i \left(\frac{B}{3} - L_i\right), \tag{5.1}$$

without making the theory anomalous. This gives electric charge ϵ_i to ν_i and generates a proton-electron charge difference ϵ_e. Constraints on this model and variants have been discussed in [15, 16, 17].

It is more difficult to give charge to massive neutrinos in this way, as discussed in [30]. The solar and atmospheric neutrino deficits can be explained by masses, mixing the three Standard Model neutrinos. If this is the case, lepton number and the lepton flavours are not conserved, so the neutrinos cannot acquire electric charge by the redefinition of equation (5.1). If there are three additional gauge singlet "right-handed" neutrinos *without* Majorana masses, the neutrino masses can conserve $B-L$ which would allow the three flavours to have the same charge ϵ. In this case, the observed neutrality of matter [1] implies $\epsilon < 10^{-21}$ [15]. It would be possible for ν_μ and ν_τ to have significantly larger charges than this if the observed solar and atmospheric neutrino deficits are not due to a neutrino mass matrix mixing the three Standard Model neutrinos.[5] Bounds on the electric charge of ν_τ were calculated in [18] without assuming any relation between the tau neutrino charge and the electric charge of the other two neutrinos. We do not consider this possibility.

6. Conclusion

We have updated the bounds from astrophysics, cosmology and laboratory experiments on fermions with electric charge ϵe, where $\epsilon < 1$. These "milli-charged" particles could be neutrinos or new fermions from beyond-the-Standard-Model. If milli-charged neutrinos have mass, and the Standard Model particle content is non-anomalous, the electric charge of neutrinos is constrained to be $< 10^{-21}$. The updated bounds on milli-charged particles from beyond-the-Standard-Model are presented in

[5] For instance, it has been suggested that the solar neutrino deficit could be caused by the deflection of milli-charged massless neutrinos in the Sun's magnetic field [31].

figure 1, for both the cases where there is, and there is not, a paraphoton. For masses $< 5\,\mathrm{keV}$, a fermion with electric charge $2 \times 10^{-14} < \epsilon e < 1$ is ruled out. An electric charge $\epsilon > 10^{-8}$ is ruled out up to masses $\sim$ MeV. Milli-charged particles could be possible for masses between an MeV and a TeV.

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Erratum

The results of [1] are in part based on those of [2] which following an independent calculation [3] has been found to contain a mistake related to a factor of two in the phase space for one of the components of the calculation. The mistake has already been briefly discussed and corrected in [4], and the full details of the correction to the original calculation can be found in [5]. The modifications that should be made to [1] in the light of these corrected results are that eqs. (4.8) and (4.10) should be changed to read

$$\beta\mathcal{R}_{ni}^{(T,D,C)} = \beta\mathcal{R}_0^{(C,D,C)}\, r_{ni}\,, \qquad r_{ni} = 2\beta_0^{-1}\left(-1.227C_A + 0.365C_A - 0.052n_f\right),$$

and

$$\begin{aligned}\mathcal{M} &= 1 + r_{in} + r_{ni} = 1 + \beta_0^{-1}\left(1.575C_A - 0.104n_f\right)\\ &= 1.490(1.430) \quad \text{for } n_f = 3(0)\,.\end{aligned}$$

We also point out that the analysis of the broadenings in [1] is superseded by that which appears in [6].

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